

**On the involution of the real line induced by Dyer's outer automorphism of  $\mathrm{PGL}(2, \mathbb{Z})$**

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(Joint work with Hakan Ayrar) Dyer's outer automorphism of  $\mathrm{PGL}(2, \mathbb{Z})$  induces an involution of the real line, which behaves very much like a kind of modular function. It has some striking properties: it preserves the set of quadratic irrationals sending them to each other in a non-trivial way and commutes with the Galois action on this set. It restricts to an highly non-trivial involution of the set unit of norm  $+1$  of quadratic number fields. It conjugates the Gauss continued fraction map to the so-called Fibonacci map. It preserves harmonic pairs of numbers inducing a duality of Beatty partitions of the set of natural numbers. It induces a subtle symmetry of Lebesgue's measure on the unit interval.

On the other hand, it has jump discontinuities at rationals though its derivative exists almost everywhere and vanishes almost everywhere.

In the talk, I plan to show how this involution acts on the quadratic irrationals.